## HOMEWORK 3 EXTRA EXERCISES (AND THINGS TO THINK ABOUT)

These are the top few rows of Pascal's triangle. The way one gets the rows, starting with the top 1 is as follows: for each entry, one adds the two entries above it together. Please convince yourself.


Exercise 1. Fill out the last row of the triangle in the picture. (You don't have to print the page, just copy down the last two rows if you'd like in your homework.)

Exercise 2. Expand

$$
\begin{aligned}
& (x+y)^{2} \\
& (x+y)^{3} \\
& (x+y)^{4} \\
& (x+y)^{5}
\end{aligned}
$$

Think about these questions:
What do you notice? What are the coefficients of the terms in the expansion? Do you recognize them in the triangle above?

Watch this explanation (from the Khan academy) of what you hopefully observed: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/binomial_theorem/ v/binomial-theorem-intuition

Also, here is another video explaining why those coefficients that show up in the expansion of $(x+y)^{n}$ are the combinations $\binom{n}{k}$; we sort of rushed over this explanation at the end of class. Check it out if you were confused:
https://www.khanacademy.org/math/algebra2/polynomial_and_rational/binomial_theorem/ v/binomial-theorem-part-3

Exercise 3. Compute the following coefficients. You do not need to expand the entire expression.
(a) What is the coefficient of $a^{4} b^{3}$ in $(a+b)^{7}$ ?
(b) What is the coefficient of $x^{4} y^{6}$ in $\left(2 y^{3}+5 x^{2}\right)^{4}$ ?

